

Polarization of the nuclear medium and RPA-type calculations in K^+ scattering from nuclei

J.C. CAILLON and J. LABARSOUQUE

Laboratoire de Physique Théorique[†], Université Bordeaux I
rue du Solarium, 33175 Gradignan Cedex, France

Abstract

In the calculation of the K^+ -nucleus cross sections, the coupling of the mesons exchanged between the K^+ and the target nucleons to the polarization of the Fermi sea has been taken into account. This polarization has been calculated in the one-loop approximation but summed up to all orders (RPA-type calculation). This effect is found to be rather important but does not improve the agreement with ex-

periment.

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E-mail LABARS@FRCPN11.IN2P3.FR

† Unité associée au CNRS 764

1 introduction

It is well known now for a long time that the K^+ mesons, which are the weakest of the strong-interacting probes, penetrate deeply into the high density regions of the nucleus[1, 2, 3]. In these regions, dynamical objects like the nucleons and mesons would behave differently than in vacuum.

It has been shown[4, 5] that the K^+ -nucleus cross-sections would be particularly sensitive to the in-medium properties of the σ , ω and ρ mesons whose exchange provides the dominant part of the K^+ -nucleon interaction. This sensitivity appears considering that, in the medium, the mesons behave like free ones but with density-dependent effective masses. For a better understanding of this effect, it would be interesting to go beyond such an approximation. In the framework of a nuclear matter built up of baryons and mesons, it seems reasonable to consider that, to a large extent, this dressing of the mesons can be interpreted as a coupling with excitations of the Fermi sea.

In this paper we have performed a calculation of the K^+ -nucleus cross sections taking into account the coupling of the σ , ω and ρ mesons exchanged between the K^+ and the target nucleons, to the polarization of the Fermi

sea. This polarization has been calculated in the one-loop approximation but summed up to all orders in the mesons propagators (RPA-type calculation, see fig 1).

We have analyzed these effects on the ratio R_T of $K^+ - {}^{12}\text{C}$ to $K^+ - d$ total cross sections which has been measured [6, 7, 8] from $400\text{MeV}/c$ to $900\text{MeV}/c$.

$$R_T = \frac{\sigma_{tot}(K^+ - {}^{12}\text{C})}{6 \cdot \sigma_{tot}(K^+ - d)} \quad (1)$$

As emphasized by many authors, this ratio is less sensitive to experimental and theoretical uncertainties than, for example, differential cross sections, and thus more transparent to the underlying physics.

In section 2 we present the formalism used to calculate the K^+ -nucleus cross sections and in section 3 how we have modified the kernel of the KN interaction to take into account the polarization of the medium. The results are then discussed in section 4.

2 The K^+ -nucleus cross-sections

The optical potential describing the elastic scattering of K^+ mesons from nuclei has been constructed by folding the nuclear proton and neutron densities, ρ_p and ρ_n , with the density-dependent K^+ -nucleon amplitude assumed to be the same as in infinite nuclear matter at the same density. The Fermi averaging, which contributes very weakly to the total cross section, has been neglected, but the nonlocalities and off-shell behaviour of the KN boson exchange model have been preserved. This leads in momentum space to:

$$\begin{aligned} \langle \vec{k}' | U | \vec{k} \rangle &= \int e^{i(\vec{k}' - \vec{k}) \cdot \vec{r}} [Z \langle \vec{k}' | t_{Kp}(\rho_p(\vec{r})) | \vec{k} \rangle \rho_p(\vec{r}) \\ &\quad + N \langle \vec{k}' | t_{Kn}(\rho_n(\vec{r})) | \vec{k} \rangle \rho_n(\vec{r})] d^3r \end{aligned} \quad (2)$$

The point-like proton distribution of the ^{12}C nucleus required in the present analysis is that deduced, after the proton finite-size correction has been made, from the electron-scattering charge density of Sick and McCarthy [9] and we have chosen equal n and p-distributions. For the deuteron, the optical potential has been calculated in the same way and we have used densities deduced from the Hulthén wave function[10].

The scattering amplitude is then calculated by solving a relativistic Lippman-

Schwinger type equation in momentum space

$$\langle \vec{k}' | T(E) | \vec{k} \rangle = \langle \vec{k}' | U(E) | \vec{k} \rangle + \int \frac{\langle \vec{k}' | U(E) | \vec{k}'' \rangle \langle \vec{k}'' | T(E) | \vec{k} \rangle}{E - E_A(\vec{k}'') - E_K(\vec{k}'') + i\epsilon} d^3 k'' \quad (3)$$

where $E_A(\vec{k}) = \sqrt{\vec{k}^2 + M_A^2}$ and $E_K(\vec{k}) = \sqrt{\vec{k}^2 + m_K^2}$ are the nucleus and K^+ energies. This equation is solved by partial wave decomposition, discretization and matrix inversion.

The K^+ -nucleus total cross section is deduced from the forward K^+ -nucleus elastic scattering amplitude using the optical theorem.

For the K^+ -nucleon amplitude in free space, we have used here the full Bonn boson exchange model[11] which is actually one of the more elaborate descriptions of the KN interaction. This K^+N interaction includes single particle exchange ($\omega, \rho, \sigma, \Lambda, \Sigma, Y^*$) but also fourth-order diagrams involving π and ρ exchange with N, Δ , K and K^* intermediate states. The solution B1, which provides the best agreement with experimental data, will be used here. We think that this agreement, though not perfect, is good enough to expect that the main part of the physics has been taken into account. In nuclear matter, this amplitude has been modified in order to take into account the coupling of the σ , ω and ρ mesons, whose exchange provides the dominant part of the KN interaction, to the polarization of the medium. The heavier particles

exchanged lead to very short-ranged processes less influenced by the nuclear environment.

3 The in-medium KN amplitude

The meson propagators taking into account the polarization of the medium can be written[12] (the Feynman propagators contain in addition a factor i which has been dropped here for simplicity):

$$G_{\sigma\sigma} = \frac{q^2 - m_\omega^2 - \Pi_q^\omega + \Pi_\eta^\omega}{(q^2 - m_\sigma^2 - \Pi_{\sigma\sigma})(q^2 - m_\omega^2 - \Pi_q^\omega + \Pi_\eta^\omega) + \Pi_{\sigma\omega}^2} \quad (4)$$

$$G_{\omega\omega}^{\mu\nu} = \frac{-g^{\mu\nu} + \eta^\mu \eta^\nu}{q^2 - m_\omega^2 - \Pi_q^\omega} - \eta^\mu \eta^\nu \frac{q^2 - m_\sigma^2 - \Pi_{\sigma\sigma}}{(q^2 - m_\sigma^2 - \Pi_{\sigma\sigma})(q^2 - m_\omega^2 - \Pi_q^\omega + \Pi_\eta^\omega) + \Pi_{\sigma\omega}^2} \quad (5)$$

$$G_{\sigma\omega}^\mu = \frac{-\eta^\mu \Pi_{\sigma\omega}}{(q^2 - m_\sigma^2 - \Pi_{\sigma\sigma})(q^2 - m_\omega^2 - \Pi_q^\omega + \Pi_\eta^\omega) + \Pi_{\sigma\omega}^2} \quad (6)$$

$$G_{\rho\rho}^{\mu\nu} = (-g^{\mu\nu} + \frac{\eta^\mu \eta^\nu}{\eta^2}) \frac{1}{q^2 - m_\rho^2 - \Pi_q^\rho} - \frac{\eta^\mu \eta^\nu}{\eta^2} \frac{1}{q^2 - m_\rho^2 - \Pi_q^\rho + \Pi_\eta^\rho} \quad (7)$$

where $\Pi_{\sigma\sigma}$ is the polarization in the σ channel and the polarization in the ω , $\sigma\omega$ and ρ channels have been decomposed as:

$$\Pi_{\omega\omega}^{\mu\nu} = -(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) \Pi_q^\omega + \frac{\hat{\eta}^\mu \hat{\eta}^\nu}{\hat{\eta}^2} \Pi_\eta^\omega \quad (8)$$

$$\Pi_{\sigma\omega}^\mu = \frac{\hat{\eta}^\mu}{(\hat{\eta}^2)^{1/2}} \Pi_{\sigma\omega} \quad (9)$$

$$\Pi_{\rho\rho}^{\mu\nu} = -(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})\Pi_q^\rho + \frac{\hat{\eta}^\mu \hat{\eta}^\nu}{\hat{\eta}^2}\Pi_\eta^\rho \quad (10)$$

Here η^μ which describes the uniform motion of the medium, is such that, in the nuclear matter rest frame we have $\eta^\mu = (1, 0, 0, 0)$ and we have noted

$$\hat{\eta}^\mu = \eta^\mu - \frac{(q \cdot \eta)q^\mu}{q^2} \quad (11)$$

For simplicity, the isospin indices have been omitted in the case of the ρ -meson. In the medium, the σ and the longitudinal part of the ω are strongly mixed and, consequently, the structure of the propagators (eq 4,5,6) is rather complicated. However, these propagators can be written in a form which makes the physics more apparent.

$$G_{\sigma\sigma} = \frac{m_+^2 - m_{\omega_L}^2}{m_+^2 - m_-^2} \frac{1}{q^2 - m_+^2} + \frac{m_-^2 - m_{\omega_L}^2}{m_-^2 - m_+^2} \frac{1}{q^2 - m_-^2} \quad (12)$$

$$G_{\omega\omega}^{\mu\nu} = \frac{-g^{\mu\nu} + \eta^\mu \eta^\nu}{q^2 - m_{\omega_T}^2} - \eta^\mu \eta^\nu \left(\frac{m_+^2 - m_{\sigma^*}^2}{m_+^2 - m_-^2} \frac{1}{q^2 - m_+^2} + \frac{m_-^2 - m_{\sigma^*}^2}{m_-^2 - m_+^2} \frac{1}{q^2 - m_-^2} \right) \quad (13)$$

$$G_{\sigma\omega}^\mu = \frac{-\eta^\mu \Pi_{\sigma\omega}}{m_+^2 - m_-^2} \left(\frac{1}{q^2 - m_+^2} - \frac{1}{q^2 - m_-^2} \right) \quad (14)$$

where m_{σ^*} is the σ -mass modified by the polarization in the σ channel

$$m_{\sigma^*}^2 = m_\sigma^2 + \Pi_{\sigma\sigma} \quad (15)$$

m_{ω_L} and m_{ω_T} are the ω -mass modified respectively by the longitudinal and transverse part of the polarization in the ω channel

$$m_{\omega_L}^2 = m_\omega^2 + \Pi_q^\omega - \Pi_\eta^\omega \quad (16)$$

$$m_{\omega_T}^2 = m_\omega^2 + \Pi_q^\omega \quad (17)$$

and m_+ , m_- are defined by

$$m_\pm^2 = \frac{1}{2}(m_{\sigma^*}^2 + m_{\omega_L}^2 \pm [(m_{\omega_L}^2 - m_{\sigma^*}^2)^2 - 4\Pi_{\sigma\omega}^2]^{1/2}) \quad (18)$$

Thus, we can see that, apart the transverse part of the ω not affected by the $\sigma - \omega$ mixing, the propagators appearing in the medium are those of two particles of mass m_+ and m_- . These two particles, arising from the mixing of the σ and of the longitudinal part of the ω , can couple either as a scalar or as a vector, giving rise to the three propagators $G_{\sigma\sigma}$, $G_{\omega\omega}^{\mu\nu}$ and $G_{\sigma\omega}^\mu$.

The propagator of the ρ -meson, more simple, can be written directly

$$G_{\rho\rho}^{\mu\nu} = (-g^{\mu\nu} + \frac{\eta^\mu\eta^\nu}{\eta^2}) \frac{1}{q^2 - m_{\rho_T}^2} - \frac{\eta^\mu\eta^\nu}{\eta^2} \frac{1}{q^2 - m_{\rho_L}^2} \quad (19)$$

where m_{ρ_L} and m_{ρ_T} are defined as in the case of the ω -meson

$$m_{\rho_L}^2 = m_\rho^2 + \Pi_q^\rho - \Pi_\eta^\rho \quad (20)$$

$$m_{\rho_T}^2 = m_\rho^2 + \Pi_q^\rho \quad (21)$$

The expressions for the polarizations $\Pi_{\sigma\sigma}$, Π_q^ω , Π_η^ω , $\Pi_{\sigma\omega}$, Π_q^ρ and Π_η^ρ at the one-loop approximation can be found in ref[12].

The KN amplitude used in this work (the Bonn boson exchange model),

built using the Time Dependent Perturbation Theory, is fully relativistic but not in a covariant form. Thus, to be able to take into account the polarization effects described above in the Feynman covariant theory, we have to establish a correspondance procedure between the two formalisms. We have used here the procedure already employed by Kotthoff et al[13] in the NN case, i.e. we have required that the KN meson-exchange potential in the Feynman formalism V^F be related to that in the Time Dependent Perturbation Theory V^{TD} by

$$V^F(E', \vec{p}'; E, \vec{p}) = \frac{1}{2}(V^{TD}(z = E + \omega_K) + V^{TD}(z = E' + \omega'_K)) \quad (22)$$

where $E = \sqrt{M_N^2 + \vec{p}^2}$, $E' = \sqrt{M_N^2 + \vec{p}'^2}$, $\omega_K = \sqrt{m_K^2 + \vec{p}^2}$ and $\omega'_K = \sqrt{m_K^2 + \vec{p}'^2}$. This can be realized considering the expressions obtained for the Feynman propagator (eq 12,13,14,19) which can be written

$$G = \sum_i \frac{\alpha_i}{q^2 - m_i^2} \quad (23)$$

and then performing an off-energy-shell extrapolation[13] of each term in order to obtain the corresponding two different time-ordered processes

$$G \longrightarrow G_{TD} = \sum_i \frac{\alpha_i}{2\omega_i} \left(\frac{1}{z - E - \omega'_K - \omega_i} + \frac{1}{z - E' - \omega_K - \omega_i} \right) \quad (24)$$

with $\omega_i^2 = \vec{q}^2 + m_i^2$.

4 Results and discussion

Considering that the dominant contribution to the forward K^+ -nucleus scattering amplitude and thus to the K^+ -nucleus total cross section comes from forward K^+ -nucleon scattering, we have estimated that, in such a region of small energy-momentum transfer, the calculation of the polarization at the one-loop order (see fig 1,a) should be a reasonable approximation. Therefore, we have used here the expressions obtained in that way by Celenza, Pantziris and Shakin[12]. However, in order to take into account, in addition, the main effect of the nuclear field on the nucleons Dirac spinors, we have replaced in the polarization the nucleon mass M by an effective mass M^* . The density dependence of this effective nucleon mass has been chosen in such a way we obtain at saturation a mass decreased by 15%, which is the value often considered as the most realistic[14]. More precisely, we have taken

$$M^*(\rho) = M(1 - 0.15\frac{\rho}{\rho_0}) \quad (25)$$

and we have verified that smooth variations from this linear dependence don't change significantly the results.

For consistency, the coupling constants and form-factors of the B1 Bonn interaction have been used at each vertex.

The σ and ω exchange terms in the KN interaction cannot be longer considered independently and have to be replaced by a “ σ - ω ” exchange which has a scalar, a vector, a scalar-vector and a vector-scalar part according to the character of its coupling to the K^+ and to the nucleon. The masses describing the in-medium propagation of this “ σ - ω ” meson are now m_{ω_T} , m_+ and m_- . The ω -mass in the transverse channel, m_{ω_T} , is weakly modified in nuclear matter (a few percents), and the main effect comes from the σ and ω -longitudinal exchanges which are strongly coupled. As discussed in the preceding section, the propagators appearing now in this sector are not those of the σ and ω mesons, but propagators of particles with mass m_+ and m_- different from m_σ and m_ω . For example, in the forward direction which gives the dominant contribution here, these masses are such that their real part (fig 2) are approximately 7% lower than the σ -mass for m_- and 10% higher than the ω -mass for m_+ , for densities higher than 20% of the saturation density. The imaginary parts grow from zero in vacuum to ~ 200 MeV at saturation. Moreover, the weights of the propagators of these “particles” with mass m_+ and m_- entering in the scalar, vector, scalar-vector and vector-scalar parts of the “ $\sigma - \omega$ ” exchange are varying with density. Thus, in the medium, the KN scattering amplitude will be appreciably different

than in free space.

In symmetric nuclear matter, the ρ -meson exchange contributes weakly to the KN interaction and its coupling to the polarization of the medium modifies very weakly the K^+ -nucleus forward amplitude. However, this coupling has been included here for completeness.

The K^+ - ^{12}C total cross section has been calculated using this medium-modified KN amplitude. For the calculation of the K^+ -deuteron cross section, since the densities involved are small and since in a nucleus made up of two nucleons it is not possible to excite more than one particle-hole pair, we have used the free-space KN interaction.

The ratio R_T obtained when the polarization of the nuclear medium is taken into account as indicated above, is shown fig.3, curve b. We can see that this effect is important since the R_T ratio is now, in average, $\sim 5\%$ lower than that calculated using the free-space KN interaction ($\sim 10\%$ at 400 MeV/c and $\sim 2\%$ at 900 MeV/c).

Unfortunately, the agreement between theory and experiment is not improved. However, we should claim that medium effects as elementary as particle-hole excitations, which are known to be important in the whole range of nuclear physics and which have been found to be important here, cannot

be ignored in this problem, even if the solution has to be searched elsewhere.

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Figure captions

Fig 1:a) Polarization of nuclear matter at the one-loop approximation in the σ - σ , σ - ω , ω - σ and ω - ω channels; b) RPA-sum contributing to the kernel of the KN interaction in nuclear matter arising from σ and ω exchange; the dashed lines represent σ or ω mesons indifferently.

Fig 2: Real part of the masses m_+ and m_- arising from the mixing of the σ and of the longitudinal part of the ω in nuclear matter calculated in the forward direction ($\nu = 0$, $\|\vec{q}\| = 10$ MeV/c)

Fig.3: Ratio R_T of the $K^+ - {}^{12}\text{C}$ and $K^+ - d$ total cross sections as a function of p_{lab} calculated, curve (a): with the free-space K^+N interaction, curve (b): with a density-dependent K^+N interaction taking into account the coupling of the σ , ω and ρ mesons with the polarization of nuclear matter calculated at the one-loop approximation. The experimental points are taken from ref.[7] (circles), from ref.[6] (squares) and from ref.[8] (triangles).

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